CALCULUS
WORKSHEET 1 ON PARTICLE MOTION

Work these on **notebook paper**. Use your calculator only on part (f) of problems 1. Do **not** use your calculator on the other problems. Write your justifications in a sentence.

1. A particle moves along a horizontal line so that its position at any time is given by
$s(t) = t^3 - 12t^2 + 36t,\ t \geq 0,$ where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at time t and at $t = 3$ seconds.
(b) When is the particle at rest? Moving to the right? Moving to the left? Justify your answers.
(c) Find the displacement of the particle after the first 8 seconds.
(d) Find the total distance traveled by the particle during the first 8 seconds.
(e) Find the acceleration of the particle at time t and at $t = 3$ seconds.
(f) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 8$.
(g) When is the particle speeding up? Slowing down? Justify your answers.

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2. The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

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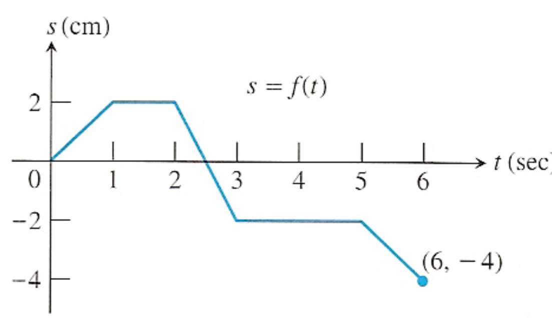
3. The figure on the right shows the position s of a particle moving along a horizontal line.
(a) When is the particle moving to the left? moving to the right? standing still? Justify your answer.
(b) For each of $v(1.5)$, $v(2.5)$, $v(4)$, and $v(5)$, find the value or explain why it does not exist.
(c) Graph the particle's velocity.
(d) Graph the particle's speed.



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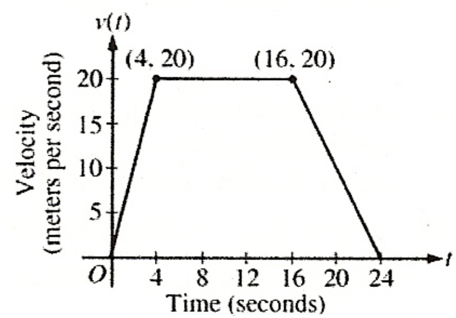
4. (2005) A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph on the right.



(a) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
(b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
(c) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

TURN->>>

5. (Modification of 2009 Form B, Problem 6)

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above.

(a) Use data from the table to estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(c) Based on the values in the table, what is the smallest number of instances at which the velocity $v(t)$ could equal $-9\,\text{m}/\text{sec}$ on the interval $0 < t < 40$? Justify your answer.

Answers to Worksheet 1 on Particle Motion

1. (a) $3t^2 - 24t + 36, \ -9\text{m}/\sec$

(b) At rest at $t = 2$ because $v(t) = 0$ there. Moving right for [0, 2) and $(6, \infty)$ because $v(t) > 0$.

Moving left for (2, 6) because $v(t) < 0$.

(c) 32 meters

(d) 96 meters

(e) $6t - 24, \ -6\text{m}/\sec^2$

(f) Graph

(g) Speeding up on (2, 4) because vel. and acc. are both neg. there and on $(6, \infty)$ because vel. and acc. are both pos. there. Slowing down on [0. 2) because vel. is pos. and acc. is neg. and on (4, 6) because vel. is neg. and acc. is pos.

2. D

3. (a) Moving left on (2, 3) and (5, 6) because $v(t) < 0$. Moving right on (0, 1) because $v(t) > 0$.

Standing still on (1, 2) and (3, 5) because $v(t) = 0$ there.

(b) 0, - 4, 0, dne because graph of s has a sharp turn there

(c) and (d) Graphs

4. (2005 AB 5)

(a) $v'(4)$ does not exist because the graph of $v(t)$ has a sharp turn at $t = 4$.

$v'(20) = -\frac{5}{2}\text{m}/\sec^2$.

(b) $a(t) = \begin{cases} 5, \ 0 < t < 4 \\ 0, \ 4 < t < 16 \\ -\frac{5}{2}, \ 16 < t < 24 \end{cases}$

(c) Ave. rate of change = $-\frac{5}{6}\text{m}/\sec^2$. No, the MVT does not apply for $8 < c < 20$ because the graph of $v(t)$ is not differentiable at $t = 16$.

5. (2009 Form B, Problem 6)

(a) $\frac{11}{8}\frac{\text{m}}{\sec^2}$

(b) The particle changes direction on (8, 20) because $v(8) = 5$ and $v(20) = -10$. The particle also changes direction on (32, 40) because $v(32) = -4$ and $v(40) = 7$.

(c) $v(t)$ must equal $-9\frac{\text{m}}{\sec}$ at least two times on (0, 40). Since $v(t)$ is differentiable, it must be continuous. $v(8) = 5, \ v(20) = -10,$ and -9 lies between 5 and – 10 so $v(t)$ must equal – 9 for some t between 8 and 20. Similarly, since $v(20) = -10, \ v(25) = -8,$ and -9 lies between – 10 and – 8 so $v(t)$ must equal – 9 for some t between 20 and 25 by the IVT>

CALCULUS
WORKSHEET 2 ON PARTICLE MOTION

Work these on **notebook paper**. Use your calculator on problems 1 - 5, and give decimal answers correct to **three** decimal places. Write your justifications in a sentence.

1. A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by $s(t) = -t^3 + 7t^2 - 14t + 8$, where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at any time t and when $t = 2$.
(b) Find the acceleration of the particle at any time t and when $t = 2$.
(c) When is the particle at rest? When is moving to the right? To the left? Justify your answers.
(d) Find the displacement of the particle during the first two seconds.
(e) Find the total distance traveled by the particle during the first two seconds.
(f) Are the answers to (d) and (e) the same? Explain.
(g) When is the particle speeding up? Slowing down? Justify your answers.

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2. The position of a particle at time t seconds, $t \geq 0$, is given by $s(t) = t^2 - \sin t,\ 0 \leq t \leq 3$, where t is measured in seconds and s is measured in meters. Find the particle's acceleration each time the velocity is zero.

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3. A particle's velocity at time t seconds, $t \geq 0$, is given by $v(t) = \cos\left(t^2\right) + t,\ 0 \leq t \leq 2$, where t is measured in seconds and v is measured in meters/second. Find the velocity of the particle each time the acceleration is zero.

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4. (2004) A particle moves along the y-axis so that its velocity at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$.

(a) Find the acceleration of the particle at time $t = 2$.
(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.
(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

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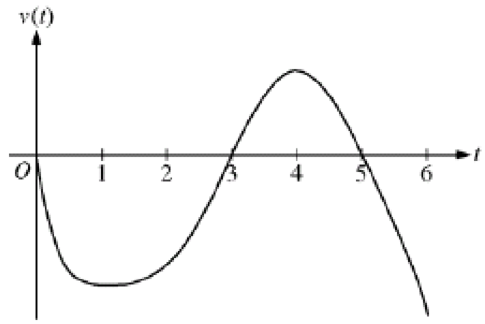
5. (Modification of 2005 Form B, Problem 3)
A particle moves along the x-axis so that its velocity at time t, for $0 \leq t \leq 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$.

(a) Find the acceleration of the particle at time $t = 4$.
(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left? Justify your answer.
(c) Find the average rate of change of $v(t)$ on $1.5 \leq t \leq 3.2$.

TURN->>>

6. (Modification of 2008, Problem 4)



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$.

(a) On the interval $3 < t < 4$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(b) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(c) During what intervals, if any, is the acceleration of the particle negative? Justify

Answers to Worksheet 2 on Particle Motion

1. (a) $-3t^2+14t=14,\quad 2\,\text{m}/\text{sec}$

   (b) $-6t+14,\ 2\,\text{m}/\text{sec}^2$

   (c) At rest at $t = 1.451$ and $t = 3.215$ because $v(t) = 0$ there. Moving left for [0, 1.451) and $(3.215,\ \infty)$ because $v(t) < 0$. Moving right for (1.451, 3,215) because $v(t) > 0$.

   (d) – 8 m

   (e) 9.262 m

   (f) No, the displacement and distance are not the same because the particle changed direction at $t = 1.451$.

   (g) Slowing down on (0, 1.451) and (2.333, 3.215) because vel. and acc. have opposite signs. Speeding up on (1.451, 2.333) and $(3.215,\ \infty)$ because vel. and acc. have the same sign.

2. $a(0.45018...)=2.435\,\text{m}/\text{sec}^2$

3. 1.600 $\frac{\text{m}}{\text{sec}}$, 0.730 $\frac{\text{m}}{\text{sec}}$

4. (a) – 0.133

   (b) – 0.436. Speed is increasing at $t = 2$ because $v(t)$ and $a(t)$ are both negative.

   (c) $v(t) = 0$ when $t = 0.443$. This is the only critical number. $v(t) > 0$ for (0, 0.443) and $v(t) < 0$ for $(0.443,\ \infty)$ so the particle reaches its highest point at $t = 0.443$.

5. (a) 0.714

(b) The particle changes direction at $t = 1$ and at $t = 2$ because $v(t)$ changes from positive to negative or vice versa there. The particle travels to the left on (2, 3) because $v(t) < 0$ there.

(c) 0.929

6. (2008)

(a) On (3, 4) $v(t) > 0$ and $v(t)$ is increasing so $v'(t) = a(t) > 0.$ Therefore, the speed is increasing on (3, 4).

(b) On (2, 3), $v(t) < 0$ and $v(t)$ is increasing so $v'(t) = a(t) > 0.$ Therefore, the speed is decreasing on (2, 3).

(c) The acceleration is negative on (0, 1) and (4, 6) because the velocity is decreasing there.