Derivatives – Quick Derivatives

A Clarification

- Taking the derivative of $\dfrac{x^2}{x}$

Derivatives – Quick Derivatives

Higher Order Derivatives

- You can take multiple derivatives of a single function
- Notation…

Derivatives – Quick Derivatives

Higher Order Derivatives

- You can take multiple derivatives of a single function

Example- $f(x) = x^3 + 5x + 2$

Derivatives – Quick Derivatives

Higher Order Derivatives

- You can take multiple derivatives of a single function

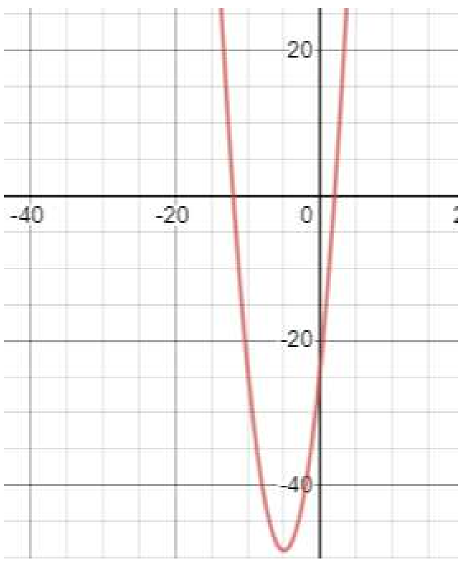
Example- $y = x^2 + 10x - 24$

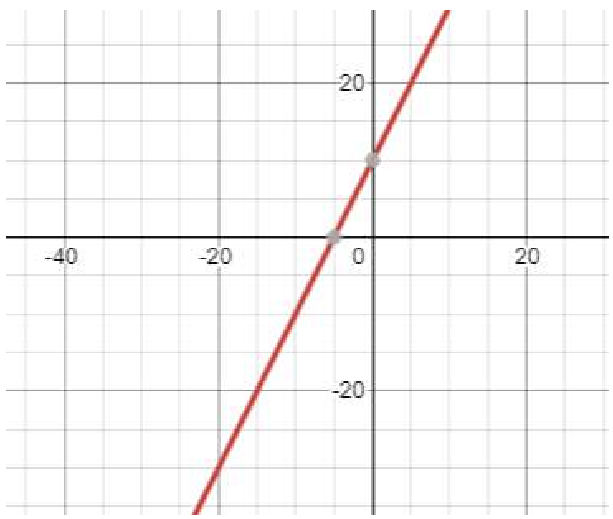
Derivatives – Quick Derivatives

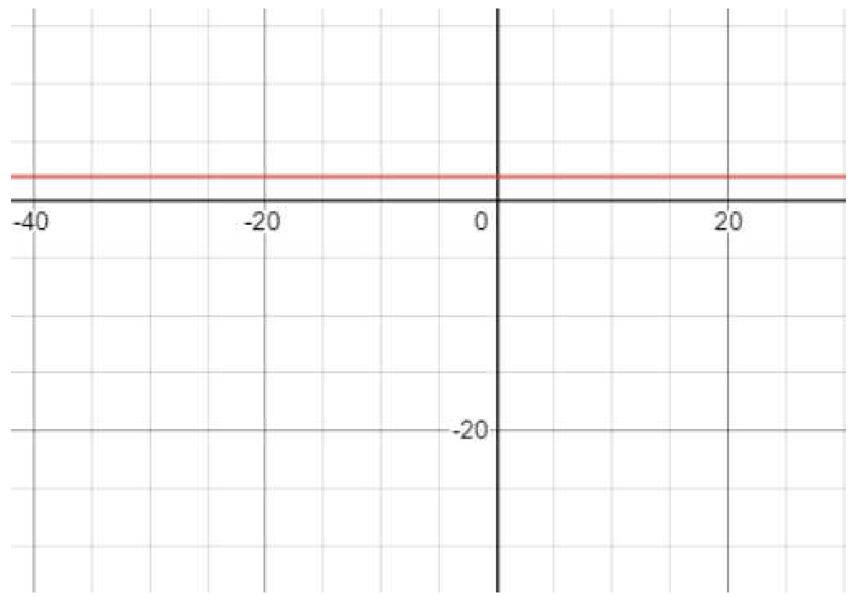
Higher Order Derivatives

- Context…
- Each derivative is a RATE OF CHANGE

Example- $y = x^2 + 10x - 24$







Derivatives – Word problems

Working with Derivatives as Rate of Change…

- Most common word problem
- SVA
- S- Position (why is it s not p?)
- V- Velocity
- A- Acceleration
- J- Jerk

Derivatives – Word problems

Working with Derivatives as Rate of Change…

- Speed up Vs Slow down
  - Idea…
  - Scenario 1
  - $f'(2)=7$, $f''(2)=5$
  - Scenario 2
  - $f'(2)=-5$, $f''(2)=14$
  - Mnemonic Bird

Derivatives – Word problems

Working with Derivatives as Rate of Change…

- Caution…
  - Distance VS Displacement
    - We will worry about this later
  - Speed Vs Velocity.

Derivatives – As Rates of Change

A Clarification

This is an application of derivative

- The key words are RATE OF CHANGE
- AVERAGE rate of change vs INSTANTANEOUS rate of change

Average

$$\frac{\text{End-Start}}{\text{Total time}}$$

Key word: AVERAGE

Instantaneous

- derivative function evaluated

Key word: AT/IN/WHEN…

Derivatives – As Rates of Change

A Clarification

Examples

Average:

- Mr. Mori was driving… what was the average velocity?
- People were voting for… On average, how many ballots were…
- Water was filling a… on average, how quickly was the height rising?

Instantaneous:

- Mr. Mori was driving… how fast was he driving 5 minutes…?
- People were voting for… When was ballots coming in the quickest?
- Water was filling... 3 minutes after it started,…?

Derivatives – As Rates of Change

Examples

- The equation $s = (1/2)gt^2$ is used to find the distance from rest of an object in freefall, where s is distance, g is the gravitational *constant* and t is time from release. At sea level the gravitational constant is $g = 9.8\text{m}/\sec^2$

1) If Mr. Mori were to drop a ball 30 feet from the ground, (assuming near sea level)
   A. What would the ball's speed and acceleration be 1 second after drop?
   B. What would the ball's speed and acceleration be 3 seconds after drop?
   C. Sometimes this equation is written as $h = -4.9t^2$
      Explain WHY this variation exists.
   D. PROVE that the ball is always speeding up

Derivatives – As Rates of Change

Examples

A blast blows a heavy rock straight up so that the height can be found using $h = 160t - 16t^2$ where h is height and t is time after blast.

1) How high does the rock go?
2) What is the speed when the rock is 256 ft above the ground?
3) Does this equation hold for any t? If not, what t values does this NOT work for and why?

Derivatives – As Rates of Change

Examples

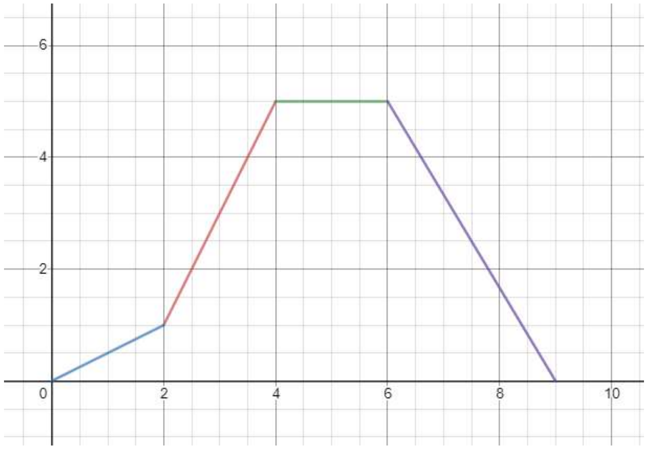
A blast blows a heavy rock straight up so that the height can be found using $h = 160t - 16t^2$ where h is height and t is time after blast.

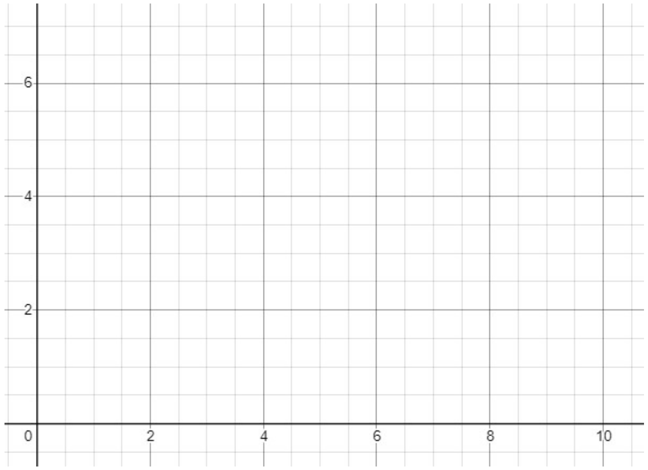
1) How high does the rock go?
2) What is the speed when the rock is 256 ft above the ground?
3) Does this equation hold for any t? If not, what t values does this NOT work for and why?
4) What is the average velocity of the rock from the time of the blast to t=10?

Derivatives – As Rates of Change

Graphing

Given this graph of a particles position on the y axis, where y is position x is time, please graph the derivative function then it's acceleration function





Derivatives – Quick Derivatives

Summary

- Be careful of instantaneous vs average rate of change
  - Derivative functions = instantaneous
  - Slope = average
- Speed vs Velocity
- Classic problems
  - Particle motion
  - Free fall
  - Population growth
  - Depth/height of water in a draining/filling container
- Make sure to know what the context is!
- Graphing
  - Remember what derivatives represent!